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Optimum temperatures in a shell and tube condenser with respect to exergy

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Abstract

This paper focuses on evaluation of the optimum cooling water temperature during condensation of saturated water vapor within a shell and tube condenser, through minimization of exergy destruction. First, the relevant exergy destruction is mathematically derived and expressed as a function of operating temperatures and mass flow rates of both vapor and coolant. The optimization problem is defined subject to condensation of the entire vapor mass flow and it is solved based on the sequential quadratic programming (SQP) method. The optimization results are obtained at two different condensation temperatures of 46 °C and 54 °C for an industrial condenser. As the upstream steam mass flow rates increase, the optimal inlet cooling water temperature and exergy efficiency decrease, whereas exergy destruction increases. However, the results are higher for optimum values at a condensation temperature of 54 °C, compared to those when the condensation temperature is 46 °C. For example, when the steam mass flow rate is 1 kg/s and the condensation temperature increases from 46 °C to 54 °C, the optimal upstream coolant temperature increases from 16.78 °C to 25.17 °C. Also, assuming an ambient temperature of 15 °C, the exergy destruction decreases from 172.5 kW to 164.6 kW. A linear dependence of exergy efficiency on dimensionless temperature is described in terms of the ratio of the temperature difference between the inlet cooling water and the environment, to the temperature difference between condensation and environment.

Keywords: Heat transfer; Temperature; Condensation; Optimization; Exergy; Shell and tube condenser

1. Introduction

Exergy is a measure of the departure of the state of a system from that of the environment. It can be defined as the maximum obtainable work from the combination of the system and environment. Unlike energy, exergy is not conserved, indeed, it is destroyed by irreversibilities. The exergy destruction during a process is proportionally related to the entropy generation due to these irreversibilities. Dincer [1] has conceptually discussed exergy from several perspectives and introduced the exergy analysis method as a useful tool for furthering the goal of more efficient use of energy resources.

Bejan [2] demonstrated the use of irreversibility as a criterion for evaluation of the efficiency of a heat exchanger. The purpose was to minimize the wasted energy by optimum design of fluid passages in a heat exchanger. In his work, the interrelationship between the losses caused by heat transfer across the stream-to-stream, due to differences in temperatures and losses caused by fluid friction, was studied.

Naterer [3] examined a numerical formulation involving the Second Law of Thermodynamics in the analysis of phase change problems with fluid flow. The discretized entropy transport equation and entropy boundary conditions are described for solid–liquid systems. This Second Law formulation can provide an effective enhancement for accurate simulations in phase change problems with fluid flow. Naterer [4] applied this formulation to problems dealing with transient heat conduction, species transport,

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Nomenclature

A	Jacobian of the constraints
A_{eff}	effective heat transfer area, m^2
a	equality constraint
c_p	specific heat, kJ/kg K
e	specific exergy flow kJ/kg
$\dot{E}_d$	exergy destruction rate, kW
$\dot{E}_{in}$	input exergy rate, kW
$\dot{E}_o$	output exergy rate, kW
f	objective function
h	enthalpy, kJ/kg
h_{fg}	condensation latent heat, kJ/kg
L	Lagrangian function, Eq. (20)
$\dot{m}$	mass flow rate, kg/s
P	pressure, kPa
Q	total heat transfer rate, kW
$\dot{Q}_j$	heat transfer rate at the location j on the boundary, kW
R	gas constant, kJ/kg K
R_m	ratio of cooling water mass flow rate to the upstream steam mass flow rate
s	entropy, kJ/kg K
s_{fg}	latent entropy, kJ/kg
T	temperature, °C
T_j	instantaneous temperature, °C
T_o	dead state/environment temperature, °C
T_{sat}	saturation temperature at the location j on the boundary, °C
U	overall heat transfer coefficient, W/m^2 K
W	Hessian of the Lagrangian
$\dot{W}_{cv}$	energy transfer rate by work, kW
X	slack variable defined in Eq. (34)
x_k	search point at kth iteration

Greek symbols

ΔT_{lm}	logarithmic mean temperature, °C
θ	dimensionless temperature, Eq. (36)
η_{ex}	exergy efficiency
δ	variable vector of quadratic problem in Eq. (28)
λ	Lagrange multiplier

Subscripts

c	coolant
cond	condensat
g	non-condensable gas
i	inlet
o	outlet
v	vapor

and melting and solidification with natural convection. He showed the importance of the Second Law as an effective complement to the discretized conservation equations in phase change computations with fluid flow.

Adeyinka and Naterer [5] presented an entropy-based procedure to assess the solution accuracy in heat transfer problems with fluid flow using the second law of thermodynamics. The procedure was implemented by a control-volume-based finite-element formulation for discrete equations arising from the conservation laws and the second law. The study involved a comparison of the local entropy production rates computed from two forms of the discretized entropy equation. The results demonstrated that the second law of thermodynamics provides an effective complement in the numerical prediction of heat transfer problems with fluid flow.

In a past study of Johannessen et al. [6], it is theoretically proven that the entropy production due to heat transfer in a heat exchanger is a minimum, when the local entropy production is constant throughout all parts of the system. A new design strategy, involving losses due to fluid and heat transfer irreversibilities that lead to production of entropy, has been recently presented by Lerou et al. [7] and applied to the thermal design of a counter-flow heat exchanger through minimization of entropy generation. Ogulata et al. [8,9] studied a manufactured plate-type cross-flow heat exchanger through the minimum entropy generation number with respect to the second law of thermodynamics. They [9] stated that the minimum entropy generation number depends on parameters such as optimum flow path length, dimensionless mass velocity, dimensionless heat transfer area and dimensionless heat transfer volume. Ko [10] analyzed numerically the steady laminar forced convection and entropy generation in a helical coil with a constant wall heat flux and water as working fluid. According to the minimum entropy generation principle and the second law, the analysis of optimal Reynolds number for the helical coil flow with a constant wall heat flux was carried out. The optimal Reynolds numbers were found to be related to the wall heat flux. The optimal Reynolds number was chosen as the flow operating condition so that the thermal system could have the least irreversibility and best exergy utilization.

Lin et al. [11] conducted a second law analysis for a saturated FC-22 vapor flowing through horizontal cooling tubes in a condenser. They reported the existence of an optimal cooling temperature that generates a minimum of entropy for a given duty parameter, which depends strongly upon many process parameters, e.g. mass flow rate and tube geometry. Li and Yang [12] performed a thermodynamic analysis of a saturated vapor flowing slowly onto and condensing on an elliptical cylinder. The authors showed how a geometrical parameter, ellipticity, affects entropy generation during a film-wise condensation process. They also obtained an expression for the minimum entropy generation in laminar film condensation. Adeyinka and Naterer [13] investigated the physical significance of entropy production and a resulting optimization correlation for laminar film condensation on a flat plate.

Utilization of the exergy method in heat exchangers has been performed by various researchers. Akpinar [14] studied experimentally the effects on heat transfer, friction factor and dimensionless exergy loss, by mounting helical (spring shaped) wires of different pitch in the inner pipe of a double pipe heat exchanger. The effects of process parameters such as the mass flow rate and temperature on the entropy generation and exergy loss were theoretically and experimentally investigated by Naphon [15] for a horizontal concentric tube heat exchanger. In the past work of San and Jan [16] on a wet cross-flow heat exchanger, the effectiveness, exergy recovery factor and second law efficiency of the wet heat exchanger were individually defined and numerically determined for various operating conditions. Additionally, the exergy-based thermoeconomic methodology has been used in different applications (e.g., [17,18]), for optimization purposes.

Shell side condensation is relevant to many important applications, both in power and process industries. In the present work, the exergy destruction and exergy efficiency for condensation of a vapor in a shell and tube condenser are modeled. Then, the optimization problem is formulated to obtain the optimal upstream coolant temperature, which results in the minimum (maximum) exergy destruction (efficiency) for a given heat transfer area of heat exchanger. The sequential quadratic programming (SQP) method is utilized to solve the optimization problem for a typical condenser. Results are discussed and presented for an application at different vapor mass flow rates.

2. Formulation for exergy

The objective of this section is to derive the exergy destruction and exergy (second law) efficiency for condensation of a pure vapor within a shell and tube condenser. The steady-state exergy rate balance for a control volume can be written as [19]

$$0 = \sum_j \left(1 - \frac{T_o}{T_j}\right)\dot{Q}_j - \dot{W}_{cv} + \sum_i \dot{m}_i e_i - \sum_o \dot{m}_o e_o - \dot{E}_d \tag{1}$$

The term $\dot{Q}_j$ represents the time rate of heat transfer at the location on the boundary where the instantaneous temperature is T_j. The term $\dot{W}_{cv}$ represents the time rate of energy transfer by work, other than flow work. The term $\dot{m}e$ accounts for the time rate of exergy transfer accompanying mass flow and flow work, with subscripts i and o representing the inlet and outlet respectively. The specific flow exergy, e, is evaluated using Eq. (2),

$$e = h - h_o - T_o(s - s_o) + \frac{V^2}{2} + gz \tag{2}$$

where h and s denote, respectively, enthalpy and entropy of the system and h_o and s_o are the values of the same properties, if the system was at the dead state. Also, T_o refers to the dead state (environment) temperature.

In Eq. (1), the term $\dot{E}_d$ accounts for the time rate of exergy destruction due to the irreversibilities within the control volume. For a condenser as shown in Fig. 1, we have

$$\dot{E}_{in} = \sum_i \dot{m}_i e_i = \dot{m}_{v1} e_{v1} + \dot{m}_{c1} e_{c1} \tag{3}$$

and

$$\dot{E}_o = \sum_o \dot{m}_o e_o = \dot{m}_{c2} e_{c2} + \dot{m}_{cond} e_{cond} \tag{4}$$

where $\dot{m}_{c1} = \dot{m}_{c2} = \dot{m}_c$. Subscripts 1 and 2 refer to the inlet and outlet, respectively. Thus, with $\dot{Q}_j = \dot{W}_{cv} = 0$, Eq. (1) can be written as follows:

$$(\dot{m}_{v1} e_{v1} + \dot{m}_c e_{c1}) - (\dot{m}_c e_{c2} + \dot{m}_{cond} e_{cond}) = \dot{E}_d \tag{5}$$

With respect to the mass conversation of vapor,

$$\dot{m}_{v1} = \dot{m}_{cond} = \dot{m}_v \tag{6}$$

Eq. (5) is rearranged as

$$\dot{m}_v(e_{v1} - e_{cond}) = \dot{m}_c(e_{c2} - e_{c1}) + \dot{E}_d \tag{7}$$

The second law efficiency, i.e. exergy efficiency, can be now defined as

$$\eta_{ex} = \frac{\dot{m}_c(e_{c2} - e_{c1})}{\dot{m}_v(e_{v1} - e_{cond})} \tag{8}$$

In other words, η_{ex} is described as the ratio of the net increase in the flow exergy of cold fluid (coolant) between the inlet and outlet, to the net decrease of flow exergy of hot fluid (vapor) from the inlet to outlet.

The term $e_2 - e_1$, net change of flow exergy, is evaluated using Eq. (2) as follows, where the kinetic and potential energy terms are negligible.

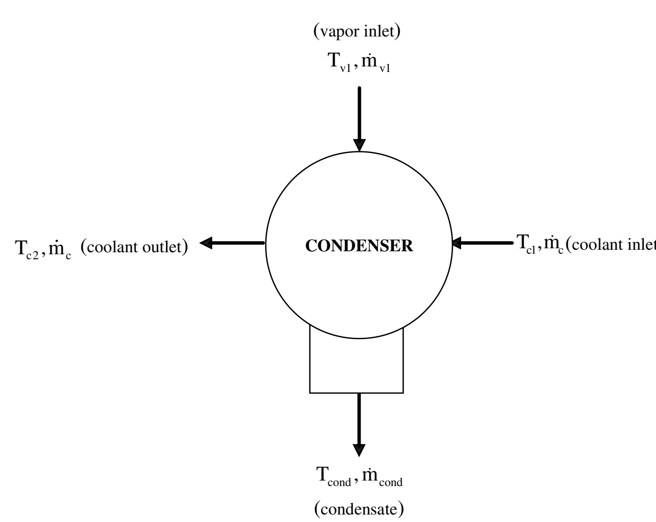

Fig. 1. Schematic of inlet and outlet flows of condenser.

$$e_2 - e_1 = h_2 - h_1 - T_o(s_2 - s_1) \quad (9)$$

Assuming a constant specific heat, c_p, the difference of enthalpy of two states of a process is

$$h_2 - h_1 = c_p(T_2 - T_1) \quad (10)$$

Also, the change of entropy between two states of a process for incompressible fluid flow can be written as

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) \quad (11)$$

Thus, the net change of flow exergy of coolant can be written as

$$e_{c2} - e_{c1} = c_{p,c}\left[(T_{c2} - T_{c1}) - T_o \ln\left(\frac{T_{c2}}{T_{c1}}\right)\right] \quad (12)$$

Also,

$$e_{v1} - e_{cond} = h_{v1} - h_{cond} - T_o(s_{v1} - s_{cond}) \quad (13)$$

The above method cannot be used to determine the difference between inlet flow exergy of steam and flow exergy of condensate. As condensation occurs at $T_{cond} \leqslant T_{v1}$, the difference between the inlet enthalpy of vapor at a temperature T_{v1} and condensate enthalpy is the sum of heat due to cooling the vapor from T_{v1} to T_{cond} and latent heat released at the condensation temperature. It may be written in the form of the following expression.

$$h_{v1} - h_{cond} = c_{p,v}(T_{v1} - T_{cond}) + h_{fg|_{T_{cond}}} \quad (14)$$

In addition, the entropy of inlet vapor, s_{v1}, may be expressed as the sum of an entropy difference due to the temperature difference $T_{v1} - T_{cond}$ at constant pressure P_{v1} and the entropy of saturated vapor at temperature T_{cond}, $s_{v|T=T_{cond}}$. Hence, the entropy difference in Eq. (13) can be written as

$$\begin{aligned} s_{v1} - s_{cond} &= \Delta s_v + s_{v|T=T_{cond}} - s_{cond|T=T_{cond}} \\ &= c_{p,v} \ln\left(\frac{T_{v1}}{T_{cond}}\right) + s_{fg|T=T_{cond}} \end{aligned} \quad (15)$$

Substituting Eqs. (14) and (15) into Eq. (13) yields

$$\begin{aligned} e_{v1} - e_{cond} &= c_{p,v}(T_{v1} - T_{cond}) + h_{fg|T=T_{cond}} \\ &\quad - T_o\left[c_{p,v} \ln\left(\frac{T_{v1}}{T_{cond}}\right) + s_{fg|T=T_{cond}}\right] \end{aligned}$$

or

$$\begin{aligned} e_{v1} - e_{cond} &= c_{p,v}\left[(T_{v1} - T_{cond}) - T_o \ln\left(\frac{T_{v1}}{T_{cond}}\right)\right] \\ &\quad + h_{fg|T=T_{cond}} - T_o s_{fg|T=T_{cond}} \end{aligned} \quad (16)$$

Note that h_{fg} and s_{fg} are dependent on the saturation temperature. Therefore, the exergy destruction, Eq. (7), and exergy efficiency, Eq. (8), can be expressed as the following functions of the inlet and outlet temperatures and mass flow rates of streams.

$$\begin{aligned} \dot{E}_d &= \dot{m}_v\left\{c_{p,v}\left[(T_{v1} - T_{cond}) - T_o \ln\left(\frac{T_{v1}}{T_{cond}}\right)\right] + h_{fg|T=T_{cond}} \right. \\ &\quad \left. - T_o s_{fg|T=T_{cond}}\right\} - \dot{m}_c c_{p,c}\left[(T_{c2} - T_{c1}) - T_o \ln\left(\frac{T_{c2}}{T_{c1}}\right)\right] \end{aligned} \quad (17)$$

$$\eta_{ex} = \frac{\dot{m}_c c_{p,c}\left[(T_{c2} - T_{c1}) - T_o \ln\left(\frac{T_{c2}}{T_{c1}}\right)\right]}{\dot{m}_v\left\{c_{p,v}\left[(T_{v1} - T_{cond}) - T_o \ln\left(\frac{T_{v1}}{T_{cond}}\right)\right] + h_{fg|T=T_{cond}} - T_o s_{fg|T=T_{cond}}\right\}} \quad (18)$$

3. Optimization procedure

The objective of this section is to formulate the optimization problem for condensation of steam in a typical shell and tube condenser, and then present the resulting optimization values. Table 1 presents a set of data [20] taken from a TEMA ‘E’ shell and tube condenser, which has a nearly standard industrial design. The condenser has a heat exchange area of 30 m^2. It is 0.438 m in diameter and 2.438 m long.

Fig. 2 illustrates the dependence of exergy efficiency of the condenser on the environment temperature at different inlet cooling water temperatures, which are obtained based on the analysis in the previous section. At a specific environment temperature, a higher inlet cooling water temperature (which is able to carry more exergy) leads to higher exergy efficiency.

Table 1
Inlet and outlet measured operating parameters

Quantity	Inlet	Outlet
Steam-air mixture temperature (°C)	125	29.30
Steam mass flow rate (kg/s)	1	0.01
Air mass flow rate (kg/s)	0.092	0.092
Cooling water temperature (°C)	10.50	20.05
Cooling water mass flow rate (kg/s)	62.5	
Condenser pressure (kPa)	18.2	

Source: Ref. [20].

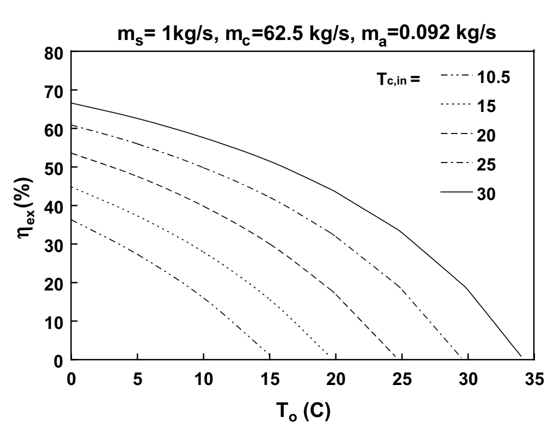

Fig. 2. Dependence of the condenser exergy efficiency on environment temperature at various upstream coolant temperatures.

In addition, in order to establish a specific η_{ex} at different environment conditions, it is required to change the inlet cooling water temperature. The graphs in Fig. 2 suggest that when $T_o - T_{c,in} \approx 4$–5 °C, the exergy efficiency of the heat exchanger is zero. The general trend is that η_{ex} decreases with ambient temperature at a constant inlet cooling water temperature, whereas it increases when the inlet cooling water temperature deviates from the environment temperature. On the other hand, as a past study [21] has shown, depending on the performance condition, increasing the inlet cooling water temperature may result in lowering the condensation rate. Fig. 3 depicts the influence of the upstream cooling water temperature on the uncondensed steam mass flow rate for two upstream temperatures of hot fluid, while the other parameters are kept constant. For example, increasing the inlet cooling water temperature from 10.5 °C to 15 °C and 20 °C at the same air leakage of 0.092 kg/s (see Table 1), results in decreasing the total condensation rate from 0.99 kg/s to 0.985 kg/s and 0.967 kg/s, respectively. In contrast, desuperheating the steam does not have a significant effect on performance parameters since the dominant source of heat is due to condensation latent heat.

This important criterion, i.e., increasing the exergy efficiency subject to condensation of the entire flow of steam, is taken into account in our optimization problem, which will be discussed in an upcoming section.

3.1. Problem definition

The aim is to establish the optimal inlet cooling water temperature through minimization of exergy destruction (or maximization of exergy efficiency) for a known condensation temperature and a given heat transfer area. It is assumed that the mass flow rates of streams are also known. In the optimization procedure, the importance of condensation of the entire vapor mass flow rate is strictly taken into account. The optimum value of the inlet cooling water temperature is established, which leads to the minimum exergy destruction, provided that the entire amount of steam condenses. Thus, the exergy destruction function, described in Eq. (17), becomes the objective function in the optimization problem, subject to the energy balance between cold and hot fluid streams as well as the governing heat transfer equation representing the heat released due to the condensation of vapor, removed to the coolant by convective heat transfer. Symbolically, we may express the optimization problem as follows:

$$\begin{aligned} &\text{minimize} && \dot{E}_d = f(T_{c1}, T_{c2}) \\ & && Q = \dot{m}_v h_{fg} = \dot{m}_c c_{pc}(T_{c2} - T_{c1}) \\ &\text{subject to} && Q = UA_{eff}\Delta T_{lm} \\ & && T_{c1} \geqslant 10\,^{\circ}\mathrm{C} \end{aligned} \tag{19}$$

where U and A_{eff} denote, respectively, the overall heat transfer coefficient and the effective heat transfer area. Also, ΔT_{lm} represents the logarithmic mean temperature defined as

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)} = \frac{(T_{cond} - T_{c2}) - (T_{cond} - T_{c1})}{\ln\left(\frac{T_{cond} - T_{c2}}{T_{cond} - T_{c1}}\right)} \tag{20}$$

The outlet cooling water temperature, T_{c2}, may be eliminated by substituting it from the first equality constraint in Eq. (19) into the corresponding equation of $\dot{E}_d$, i.e., Eq. (17) as well as Eq. (20). The lower limit for the inlet cooling water temperature is chosen to be 10 °C, which is shown by the inequality constraint. Hence, the final form of the optimization problem can be written as

$$\begin{aligned} &\text{minimize} && \dot{E}_d = f(T_{c1}) \\ &\text{subject to} && \dot{m}_v h_{fg} = UA_{eff}\Delta T_{lm} \\ & && T_{c1} \geqslant 10\,^{\circ}\mathrm{C} \end{aligned} \tag{21}$$

It can be seen that there is only one equality constraint. As mentioned previously, the aim is to find the optimum inlet cooling water temperature, by minimizing the exergy destruction ($\dot{E}_d$) of the condensation process, which satisfies the heat transfer equation (equality constraint in Eq. (21)) and must be equal or greater than 10 °C (inequality constraint in Eq. (21)). In order to solve the problem defined in Eq. (21), the sequential quadratic programming (SQP) method is utilized in this study. The next section explains the general SQP method.

3.2. Optimization method: SQP

As the name implies, sequential quadratic programming is an iterative method that solves a quadratic programming problem (QP) at each iteration [22]. Consider the following optimization problem:

$$\left.\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && a_i(x) = 0 \quad \text{for } i = 1, \ldots, p \end{aligned}\right\} \tag{22}$$

where $f(x)$ and $a_i(x)$ are continuous, with continuous second order derivatives. SQP formulates the problem at the

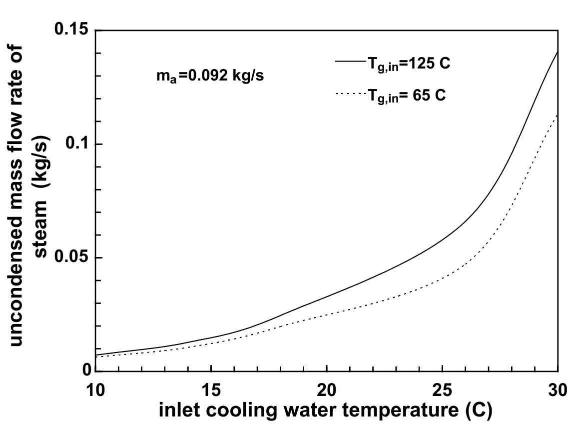

Fig. 3. Effect of inlet cooling water temperature on the rate of uncondensed steam at two different upstream temperatures of hot fluid (an air leakage of 0.092 kg/s).

current point x_k by a quadratic sub-problem and it uses the solution of this sub-problem to find the new point x_{k+1}. SQP is in a way the application of Newton's method to the Karush–Kuhn–Tucker (KKT) optimality conditions.

The Lagrangian function for this problem is

$$L(x,\lambda) = f(x) - \lambda^{\mathrm{T}} a(x) \quad (23)$$

where λ denotes the Lagrange multiplier. The Jacobian of the constraints is defined by

$$A(x)^{\mathrm{T}} = [\nabla a_1(x)^{\mathrm{T}}, \ldots, \nabla a_p(x)^{\mathrm{T}}] \quad (24)$$

If x^* is a constrained local minimizer of the problem in Eq. (22), there exists a Lagrange multiplier λ^* which satisfies

$$g(x^*) = \lambda^{*^{\mathrm{T}}} \nabla a(x^*) \quad (25)$$

where

$$g(x) = \nabla f(x) \quad (26)$$

Hence, at x^*,

$$\nabla L(x^*, \lambda^*) = 0 \quad (27)$$

At (x_k, λ_k), we try to find (x_{k+1}, λ_{k+1}) so the difference between them becomes closer to (x^*, λ^*). Using the first two terms of a Taylor series, $\nabla L(x_{k+1}, \lambda_{k+1})$ can be approximated as

$$\nabla L(x_{k+1}, \lambda_{k+1}) \approx \nabla L(x_k, \lambda_k) + \nabla^2 L(x_k, \lambda_k)\delta \quad (28)$$

where δ is the new variable vector and it is denoted by $\delta = [\delta_x, \delta_\lambda]^{\mathrm{T}}$. An improved approximation of (x^*, λ^*) is (x_{k+1}, λ_{k+1}), if

$$\nabla L(x_k, \lambda_k) = -\nabla^2 L(x_k, \lambda_k)\delta \quad (29)$$

The previous equation can be expressed as

$$\begin{bmatrix} W_k & -A_k^{\mathrm{T}} \\ A_k & 0 \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_\lambda \end{bmatrix} = \begin{bmatrix} A_k^{\mathrm{T}} - g_k \\ -a_k \end{bmatrix} \quad (30)$$

where the Hessian of the Lagrangian is denoted by $W(x,\lambda) = \nabla^2_{xx} L(x,\lambda)$.

An alternative way of expressing this formulation of the SQP is to define the following quadratic problem at (x_k, λ_k),

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\delta^{\mathrm{T}} W_k \delta + \delta^{\mathrm{T}} g_k \\ &\text{subject to} \quad A_k \delta + a = 0 \end{aligned} \quad (31)$$

From Eq. (30), it can be inferred that this problem has a unique solution that satisfies

$$W_k \delta_x + g_k = A_k^{\mathrm{T}} \lambda_{k+1} \quad (32a)$$

$$A_k \delta_x = -a \quad (32b)$$

where δ_x is a local minimizer of the quadratic problem in Eq. (31). Therefore, for the next search direction, x_{k+1} is obtained as follows:

$$x_{k+1} = x_k + \delta_k \quad (33)$$

λ_{k+1} can be also determined from Eq. (32a). Then, x_{k+1} and λ_{k+1} are used for the next iteration.

This procedure will repeat until the convergence criteria are met. In a general nonlinear optimization problem, in order to define the sub-problem, it is required to similarly linearize both equality and inequality constrains.

Our optimization problem defined in Eq. (21) includes one equality constraint and one inequality constraint. We use a slack variable to eliminate the inequality constraint as follows:

$$X^2 = T_{\mathrm{cl}} - 10 \quad (34)$$

Hence, Eq. (21) can be rewritten as

$$\left.\begin{aligned} &\text{minimize} \quad f(X^2) = \dot{E}_{\mathrm{d}} \\ &\text{subject to} \quad a(X^2) = \dot{m}_{\mathrm{v}} h_{fg} - UA_{\mathrm{eff}} \Delta T_{\mathrm{lm}} \end{aligned}\right\} \quad (35)$$

In each iteration, one must evaluate W_k, A_k, g_k and a, defined previously, to solve the corresponding quadratic problem in Eq. (31). The next section will present results from this optimization formulation.

4. Results and discussion

Exergy destruction, $\dot{E}_{\mathrm{d}}$, for the operating condition presented in Table 1 may be now calculated at a given environment temperature, T_{o}, using Eq. (17). For instance, if $T_{\mathrm{o}} = 10\,^{\circ}\mathrm{C}$ and $T_{\mathrm{o}} = 15\,^{\circ}\mathrm{C}$, then $\dot{E}_{\mathrm{d}} = 215$ kW and $\dot{E}_{\mathrm{d}} = 221$ kW, respectively. In this section, the optimization results are presented, assuming that condensation of saturated water vapor occurs without the presence of air. The condensation temperature is a key design parameter that is directly affected by the condenser pressure. Its selection depends on various factors, including technical and other factors. Table 2 gives the optimization results at two different condensation temperatures, while the ambient temperature is assumed to be 15 °C. According to this table, it seems that exergy destruction in the condenser may be significantly diminished. Comparing Tables 1 and 2 indicates that for condensation of saturated pure water vapor, the optimized value for the upstream coolant temperature in either case shown in Table 2 is considerably greater than the value in Table 1, when condensation of superheated steam occurs in the presence of air. As mentioned previously, air is a non-condensable gas that provides resistance to heat and mass transfer in the condensation process. It reduces the condensation rate, compared to the case with less air content. As shown in Fig. 3, if the temperatures were selected according to the optimized values in Table 2, the total condensation rate would decrease. In other words, in order to condense the total mass of steam in the presence of air, it is required to select a lower upstream

Table 2
Optimum points for $\dot{m}_s = 1$ kg/s and $\dot{m}_c = 62.5$ kg/s, $T_{\mathrm{o}} = 15\,^{\circ}\mathrm{C}$

T_{cond} (°C)	P (kPa)	$T_{\mathrm{c,in}}$ (°C)	$T_{\mathrm{c,o}}$ (°C)	$\dot{E}_{\mathrm{d}}$ (kW)	η_{ex}
46	10	16.78	25.9	172.5	0.226
54	15	25.17	34.2	164.6	0.407

cooling water temperature, which accompanies the lower exergy efficiency and higher exergy destruction.

Fig. 4 illustrates the variation of optimum inlet and outlet cooling water temperatures and minimum exergy efficiency, with upstream water vapor mass flow rate, when the condensation temperature is 46 °C. It can be observed that when the steam mass flow rate increases (for a given heat transfer area), the optimum cooling water temperature decreases. As mentioned before, minimization of exergy destruction is performed when the entire upstream flow rate of steam will condense. Since a higher amount of steam removes more heat due to the latent heat release to the cooling water, for a constant condenser configuration, conservation of energy implies a higher temperature difference between the condensate and cooling water. As condensation occurs at the same temperature for all cases shown in Fig. 4, therefore, a higher temperature difference may be established by reducing the inlet cooling water temperature. Furthermore, it is seen that increasing the steam mass flow rate results in a lower optimal (maximum) exergy efficiency. As mentioned previously, a higher steam mass flow rate leads to a lower optimal inlet coolant temperature. Thus, the difference between the cooling water and environment temperatures decreases. Therefore, the ability of the system to carry the exergy decreases. As explained previously (see Fig. 2), a lower exergy efficiency may result from a smaller difference between the inlet cooling water and environment temperatures. The results of exergy efficiency follow the expected physical trends in these regards.

Additionally, the effect of the environment temperature on optimum exergy efficiency is represented in Fig. 5. Based on previous discussions, since a given operating condition, e.g. $\dot{m}_s = 1$ kg/s, leads to a fixed value of the optimum cooling water temperature (see Fig. 4), increasing the ambient temperature and therefore decreasing the temperature difference between the cooling water and ambient condition results in a lower exergy efficiency, as shown in Fig. 5. This is in agreement with the results of Fig. 2, where a higher ambient temperature at a fixed process condition

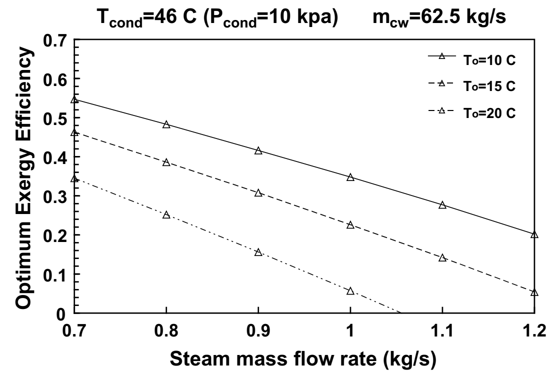

Fig. 5. Influence of environment temperature on optimum exergy efficiency at different steam mass flow rates.

leads to a lower exergy efficiency. Variations of the optimum (minimum) exergy destruction at various ambient temperatures are also shown in Fig. 6. The variation of ambient temperature has a slight effect on exergy destruction. Nevertheless, an increase in the steam mass flow rate causes more exergy destruction compared to the effect of increasing the ambient temperature. Condensation of a higher steam mass flow rate, which releases more heat, needs a lower inlet cooling water temperature (see Fig. 4). Hence, a higher amount of total heat transfer, as well as an increased temperature difference between the cooling water and condensation leads to augmentation of irreversibilities.

Further results for a condensation temperature of 54 °C are illustrated in Figs. 7–9. Comparing Figs. 7 and 4 shows that optimum cooling water temperatures are consistently higher at a higher condensation temperature. The temperature difference between condensation and coolant streams is a dominant factor in the condensation process, i.e. a certain range of this temperature difference is enough to establish an effective heat transfer rate. This fact is taken into account in the optimization procedure, when performing the procedure to find the optimal

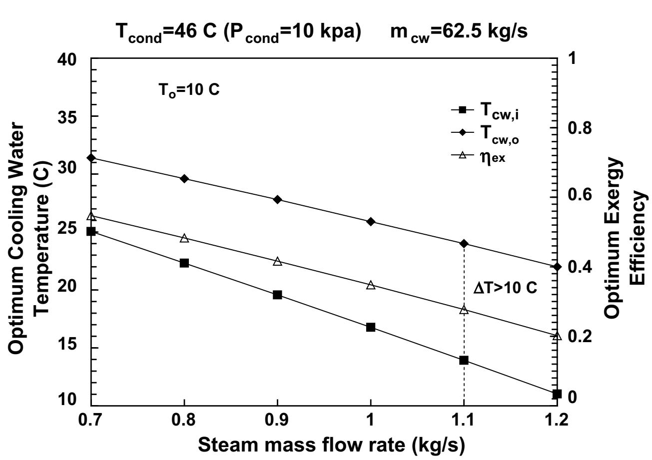

Fig. 4. Variation of optimum cooling water temperature and optimum exergy efficiency at different upstream steam mass flow rates.

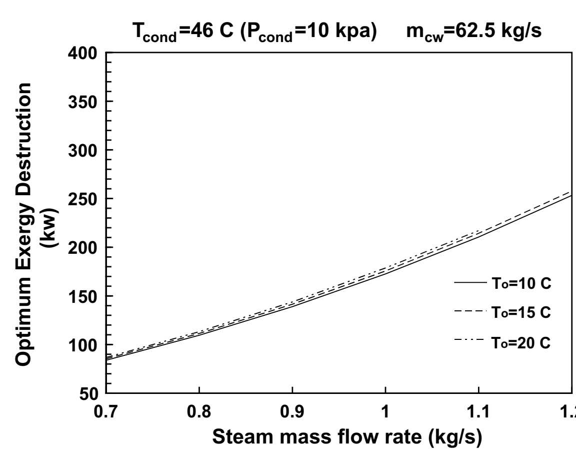

Fig. 6. Variation of minimum exergy destruction with upstream steam mass flow rates at three ambient temperatures.

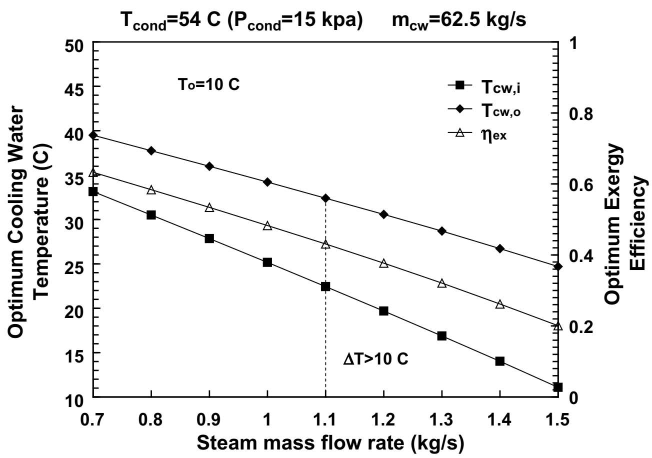

Fig. 7. Variation of optimum cooling water temperature and optimum exergy efficiency at different steam mass flow rates and condensation temperature of $T_{cond} = 54$ °C.

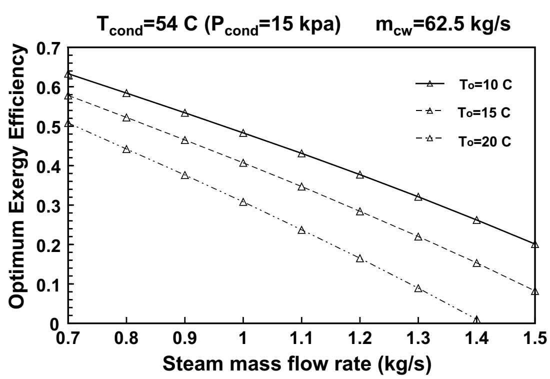

Fig. 8. Influence of environment temperature on optimum exergy efficiency at different steam mass flow rates and condensation temperature of $T_{cond} = 54$ °C.

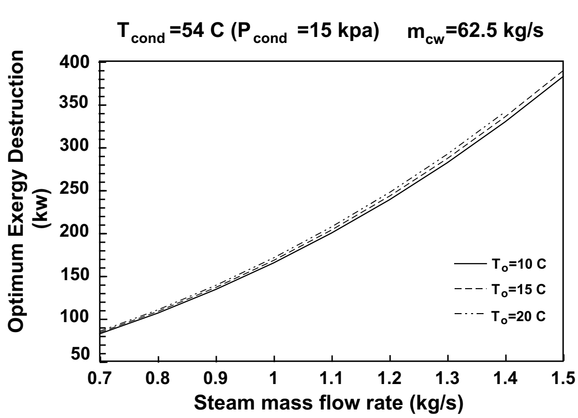

Fig. 9. Variation of minimum exergy destruction with steam mass flow rates at three different environment temperatures and condensation temperature of $T_{cond} = 54$ °C.

values for temperatures. For this reason, as the condensation temperature increases from 46 °C to 54 °C, the optimal coolant temperature has also increased in Fig. 7, compared to the relevant profiles in Fig. 4. Careful review of Figs. 4 and 7 reveals that the temperature profile in Fig. 7 has approximately shifted 8 °C upward, compared to Fig. 4; that is, an optimal inlet cooling water temperature has followed the condensation temperature as it increased from 46 °C to 54 °C. Therefore, at the same steam mass flow rate, the temperature difference between condensation and inlet cooling water is almost the same in Figs. 4 and 7. We may expect a higher exergy efficiency when the condensation temperature is 54 °C as the system deviates relatively more from the environment in this case, because of the higher cooling water temperature profile. Fig. 8 confirms this matter. Comparing it with Fig. 5 reveals significant augmentation of exergy efficiency at all environment temperatures, as the condensation temperature increases from 46 °C to 54 °C. However, comparing the related curves for exergy destruction in these two cases (Figs. 6 and 9), it does not indicate a considerable change in the amount of exergy destruction as the condensation temperature and consequently optimal coolant temperature increase. Exergy destruction is directly influenced by the total heat transfer rate, which is due to the condensation of steam. The difference between condensation and the cooling water temperature is approximately the same in Figs. 4 and 7 for a given steam mass flow rate (discussed above). Hence, condensation of the same amount of steam mass flow rate approximately results in the same exergy destruction as shown in Figs. 6 and 9.

Further studies have been performed to show that there is a systematic relation between the exergy efficiency and dimensionless temperature, θ, defined as below,

$$\theta = \frac{T_{c,in} - T_o}{T_{cond} - T_o} \quad (36)$$

The variation of optimum η_{ex} versus θ_{opt} is shown in Fig. 10. A linear relation between η_{ex} and θ can be seen, which provides a useful result for correlating results over a range of operating conditions. Particularly, operating conditions given in Table 1 from available experiments at two ambient temperatures are very close to the predicted linear trend of η_{ex} with dimensionless temperature. It is

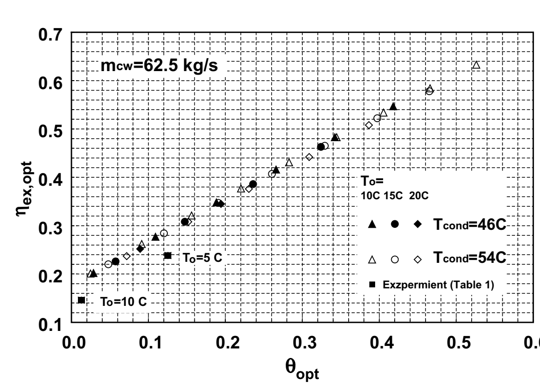

Fig. 10. Linear dependence between optimal values of exergy efficiency and dimensionless temperature.

worth noting that in the real process (recent two points), condensation of superheated steam occurs in the presence of air.

5. Conclusions

The exergy destruction and exergy efficiency of condensation of vapor in a shell and tube condenser are formulated. It is shown that they can be expressed as functions of several operating parameters, such as the inlet and outlet cooling water and condensation temperatures. The optimization problem has been formulated for a given configuration of condenser. The method of sequential quadratic programming (SQP) is utilized in this study to solve the optimization problem. The results are obtained and presented for a typical industrial condenser. Condensation of saturated steam is considered with cooling water as the coolant, through two examples that depend on the condensation temperature, which may be found frequently in actual design of a condenser. Additionally, optimization results reveal new characteristics for the cooling water, with respect to the minimization of exergy destruction of the condensation process. Further results are presented for different steam mass flow rates. It is shown that an increase in the steam mass flow rate may result in a lower optimum cooling water temperature, which consequently leads to lower exergy efficiency. Also, the optimal exergy destruction increases at higher rates of the steam mass flow.

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